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Physics

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Speed

A problem that is challenging enough for an average college student is the following. What is the slowest initial speed a boy can throw a pebble over a structure which height and length are equal to H and L respectively? Assume that the initial position of the pebble is arbitrary in the horizontal direction and equals h in the vertical one (the vertical axis points upward and its origin is the ground level).

The easy way of solving this problem consists in taking advantage of the broadly-known fact that is the following: The object thrown at the angle φ with the horizontal above the surface that is not inclined lands at the farthest point if $\varphi = 45°$. Specifically, from symmetry considerations, it follows that the initial speed is the smallest when the pebble trajectory is the one depicted on Figure 1 and $\gamma = 45°$. At the point A (see Figure 1) the vertical component of the stone velocity is equal to $v_A \sin \gamma$ whereas at the point B this component equals $-v_A \sin \gamma$.

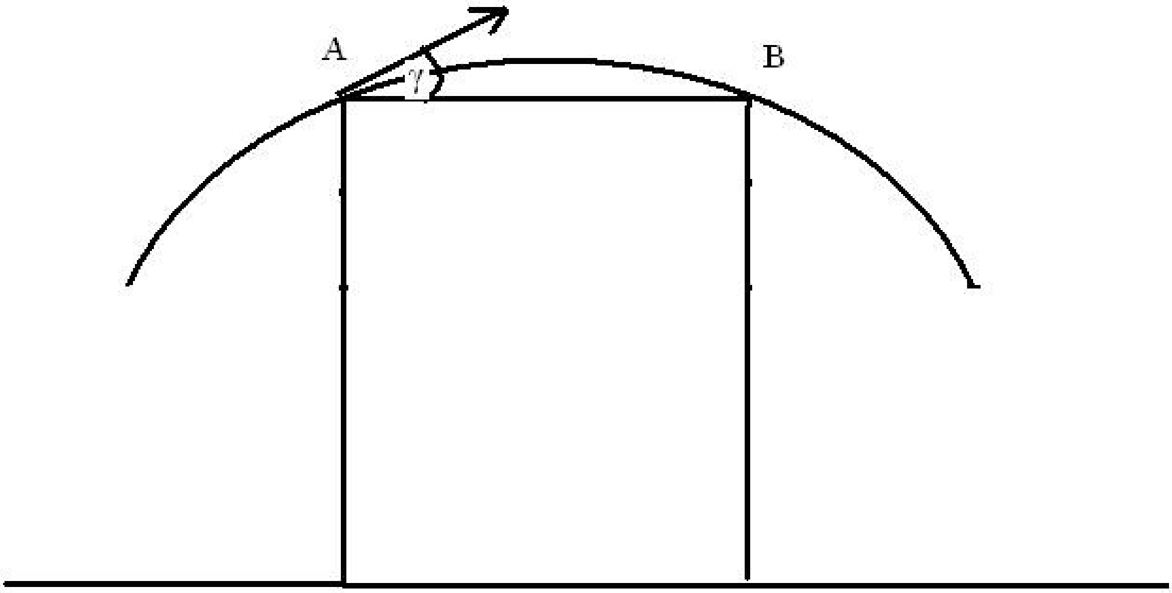

Figure 1.

The acceleration in the vertical direction is equal to -*g* where *g* is the free fall one. Hence, if t_{AB} is the time of travel of the object from point A to point B, then one can write the following:

$$t_{AB} = \frac{2v_A \sin\gamma}{g}. \quad (1)$$

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Since the horizontal component of the stone acceleration is zero, one can write the following:

$$t_{AB}\, v_A \cos\gamma = L. \quad (2)$$

From the equations (1) and (2), it follows that

$$V_A = \sqrt{Lg} \quad (3)$$

since $\gamma = 45°$. Hence, from the energy conservation and the equation (3), it follows that the smallest initial speed v_0 of the pebble that guarantees reaching the opposite side of the structure satisfies such equation:

$$gh + \frac{v_0^2}{2} = gH + \frac{gL}{2}. \tag{4}$$

From the equation (4), it follows that $v_0 = \sqrt{g(L + 2(H - h))}$